Problem I
Economic-Demographic Interrelations

Consider a neoclassical household model in which parents have Cobb-Douglas utility with respect to consumption of goods services Z_g and child services Z_c.

$$U(Z_g,Z_c) = Z_g^{1/2}Z_c^{1/2}$$

Assume that goods services are producing using only market goods ($Z_g = x_g$) and child services are produced using the following production function:

$$Z_c = \min(x_c, H_m^{1/2}+H_f^{1/2}).$$

Assume that the male wage w_m exceeds the female wage w_f, and that males and females each have 12 hours to split between working and child care. You may assume that the price of market goods used in the production of goods services and child services is 1. You may assume that in equilibrium both parents do some labor market work.

a) find an expression for the cost function for child services

b) find an expression for the shadow price of child services

c) sketch the budget constraint (Hint: the budget constraint is not a straight line)

d) solve for the optimal level of child services under the following three assumptions

(i) $w_m=2$, $w_f=1$, $v=0$

(ii) $w_m=2$, $w_f=2$, $v=0$

(iii) $w_m=3$, $w_f=1$, $v=0$

(iv) $w_m=2$, $w_f=1$, $v=1$

Note: Use the quadratic equation

$$\textit{if } ax^2+bx+c=0 \textit{ then } x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

e) Find the number of hours spent in child care for each parent in each of the cases in (d).