Economics 791
Problem Set II

Consider a two period world. In each period a woman may choose whether or not she wishes to contracept and the probability that she will give birth depends on whether or not she is using contraception as well as her fecundity. Her choice of contraception in the second period may depend on the outcome of the first period. Choices are selected in order to maximize expected utility.

Use the following notation:

- b_t is the birth outcome in period t which takes the value 1 if a birth takes place and 0 otherwise.
- c_t is the contraceptive choice in period t which takes the value 1 if contraception is used in period t and 0 otherwise.
- μ is the fecundity of a woman
- $q(\mu)$ is the proportion of the population with fecundity μ
- $U(b,c)$ is the utility of a woman who has b births and uses contraception c times over the two periods (i.e. $U(1,2)$ is the utility of a woman who used contraception in both periods but gave birth in only one period.
- $E_t U$ is the expected utility conditional on information known at the start of period t.
- $f(b_t;c_t,\mu)$ is the probability that a woman of fecundity μ with contraceptive choice c_t in period t will experience birth outcome b_t.
- CER_t is the "contraceptive efficacy rate" which is the observed probability of conception for a woman at time t who does not use contraception minus the probability for one who does.

Let

$$U(b,c) = -3*(b-1)^2 - c$$

and

$$f(1;\mu,c_t) = 1-f(0;\mu,c_t) = \mu-.4c.$$

Thus a woman with fecundity 0.7 who uses contraception will have a .3 chance of giving birth, one who does not will have a .7 chance of giving birth, and the CER = .4. The expected utility in period 2 of a woman with one birth in period 1 who uses contraception in both periods is thus

$$E_2 U = (\mu-.4)\, U(2,2) + (1-\mu+.4)\, U(1,2)$$
$$= .3\,(-3-2) + .7*(-2) = -2.9$$

and that of a woman with one birth in period 1 who uses contraception only in the first period will be

$$E_2 U = .7\,(-3-1) + .3*(-1) = -3.1$$

and thus the woman will choose to contracept.

Part I. Assume μ=.7 (i.e. q(0.7)=1) and that women know their own fecundity. Answer the following questions.

a) Do women who gave birth and did not contracept in the first period contracept in the second period?
b) Do women who did not contracept nor give birth in the first period contracept in the second period?
c) Answer questions (a) and (b) for woman who did contracept in the first period.
d) Do women contracept in the first period?
e) What is the contraceptive efficacy rate in period 2?

Part II. Assume now that the population consists of equal numbers of low fecundity women (μ_1=0.4) and high fecundity women (μ_2 =1) (i.e. q(1)=q(0.4)=0.5). Although women do not know their own fecundity they know that they have a 50% chance of being highly fecund. They also know about conditional probabilities so they can compute their probability of being highly fecund conditional on their experience in the first period (i.e. whether or not they used contraception). Answer the following questions using the attached table and/or the following equations:

$$P_1(b_1;c_1)=\sum_{\mu} q(\mu)f(b_1;c_1,\mu)$$

$$P_2(b_2;c_2,c_1,b_1)=\frac{\sum_{\mu} q(\mu)f(b_1;c_1,\mu)f(b_2;c_2,\mu)}{\sum_{\mu} q(\mu)f(b_1;c_1,\mu)}$$

$$c_1^*=\underset{c_1}{\arg\max}\sum_{b_1}\sum_{b_2}P_1(b_1;c_1)*P_2(b_2;c_2^*(c_1,b_1),c_1,b_1)U(b_1+b_2,c_1+c_2^*(c_1,b_1))$$

$$c_2^*(b_1,c_1)=\underset{c_2}{\arg\max}\sum_{b_2}P_2(b_2;c_2,c_1,b_1)U(b_1+b_2,c_1+c_2)$$

where

$P_1(b_1;c_1)$ is the probability in period 1 that a woman with contraceptive choice c_1 will have birth outcome b_1 given the distribution of fecundity at the start of period 1;

$P_2(b_2;c_2,c_1,b_1)$ is the conditional probability that a woman with contraceptive choices c_2 and c_1 who experienced a birth outcome b_1 in period 1 will experience a birth outcome b_2 in period 2, given the distribution of fecundity among women with contraceptive choice c_1 and birth outcome b_1 in period 1;

c_1^* represents the optimal action in period 1; and

$c_2^*(b_1,c_1)$ represents the optimal action in period 2 conditional on the action and birth outcome in period 1.

a) Do women who gave birth and did not contracept in the first period contracept in the second period?
b) Do women who did not contracept nor give birth in the first period contracept in the second period?
c) Answer questions (a) and (b) for woman who did contracept in the first period.
d) Do women contracept in the first period?
e) What is the probability that a woman who gave birth in the first period is highly fecund?
f) What is the contraceptive efficacy rate in period 2?
g) Why does your answer to Part IIf not equal the answer you obtained in Part Ie?

c_1	c_2	b_1	b_2	μ_1	μ_2	$f(b_1; c_1,\mu_1)$	$f(b_1; c_1,\mu_2)$	$f(b_2; c_2,\mu_1)$	$f(b_2; c_2,\mu_2)$	$P_1(b_1; c_1)$	$P_2(b_2; c_2,c_1,b_1)$	$U(b,c)$
0	0	0	0	0.4	1	0.6	0	0.6	0	0.3	0.6	-3
0	0	0	1	0.4	1	0.6	0	0.4	1	0.3	0.4	0
0	1	0	0	0.4	1	0.6	0	1	0.4	0.3	1	-4
0	1	0	1	0.4	1	0.6	0	0	0.6	0.3	0	-1
0	0	1	0	0.4	1	0.4	1	0.6	0	0.7	0.171429	0
0	0	1	1	0.4	1	0.4	1	0.4	1	0.7	0.828571	-3
0	1	1	0	0.4	1	0.4	1	1	0.4	0.7	0.571429	-1
0	1	1	1	0.4	1	0.4	1	0	0.6	0.7	0.428571	-4
1	0	0	0	0.4	1	1	0.4	0.6	0	0.7	0.428571	-4
1	0	0	1	0.4	1	1	0.4	0.4	1	0.7	0.571429	-1
1	1	0	0	0.4	1	1	0.4	1	0.4	0.7	0.828571	-5
1	1	0	1	0.4	1	1	0.4	0	0.6	0.7	0.171429	-2
1	0	1	0	0.4	1	0	0.6	0.6	0	0.3	0	-1
1	0	1	1	0.4	1	0	0.6	0.4	1	0.3	1	-4
1	1	1	0	0.4	1	0	0.6	1	0.4	0.3	0.4	-2
1	1	1	1	0.4	1	0	0.6	0	0.6	0.3	0.6	-5